

## 2. FUNCTIONS



- Function, Domain, Co-domain, Range
- Types of functions – One-one, Onto
- Representation of function
- Evaluation of function
- Fundamental types of functions
- Some special functions



### 2.1 FUNCTION

**Definition :** A function  $f$  from set  $A$  to set  $B$  is a relation which associates each element  $x$  in  $A$ , to a unique (exactly one) element  $y$  in  $B$ .

Then the element  $y$  is expressed as  $y = f(x)$ .

$y$  is the image of  $x$  under  $f$

If such a function exists, then  $A$  is called the **domain** of  $f$  and  $B$  is called the **co-domain** of  $f$ .

For example,

Examine the following relations which are given by arrows of line segments joining elements in  $A$  and elements in  $B$ .

1)

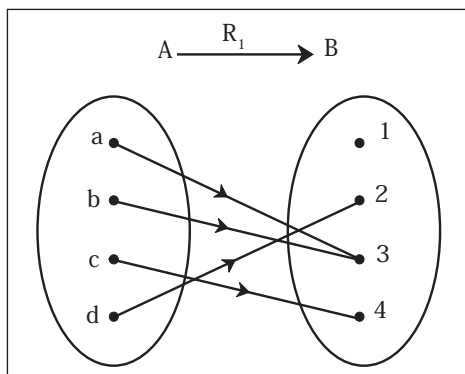


Fig. 2.1

$R_1$  is a well defined function.

2)

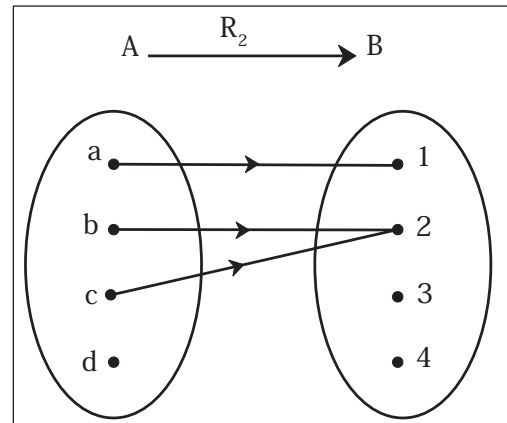


Fig. 2.2

$R_2$  is not a function because element  $d$  in  $A$  is not associated to any element in  $B$ .

3)

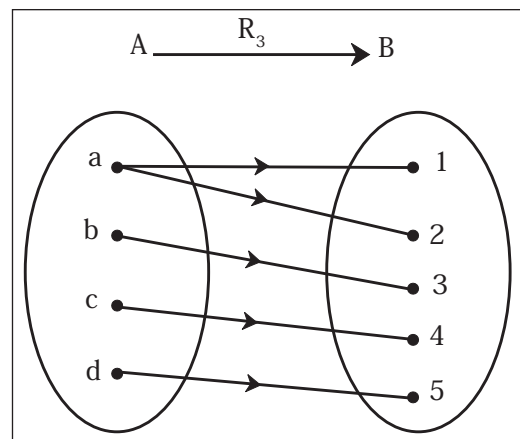


Fig. 2.3

$R_3$  is not a function because element  $a$  in  $A$  is associated to two elements in  $B$ .

The relation which defines a function  $f$  from domain  $A$  to co-domain  $B$  is often given by an algebraic rule.

#### 2.1.1 Types of function

##### One-one or One to one or Injective function

**Definition :** A function  $f: A \rightarrow B$  is said to be one-one if distinct elements in  $A$  have distinct images

in B. The condition is also expressed as

$$a \neq b \Rightarrow f(a) \neq f(b)$$

### Onto or Surjective function

**Definition:** A function  $f: A \rightarrow B$  is said to be onto if every element  $y$  in B is an image of some  $x$  in A

The image of A can be denoted by  $f(A)$ .

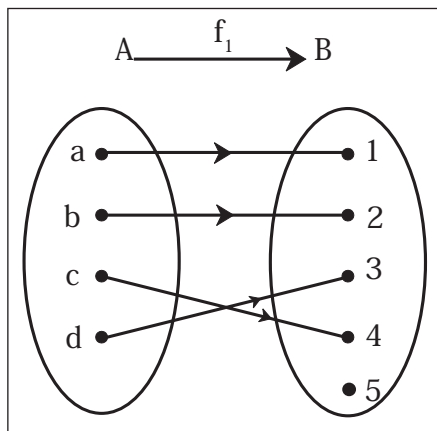
$$f(A) = \{y \in B \mid y = f(x) \text{ for some } x \in A\}$$

$f(A)$  is also called the **range** of A.

Note that  $f: A \rightarrow B$  is onto if  $f(A) = B$ .

**For example,**

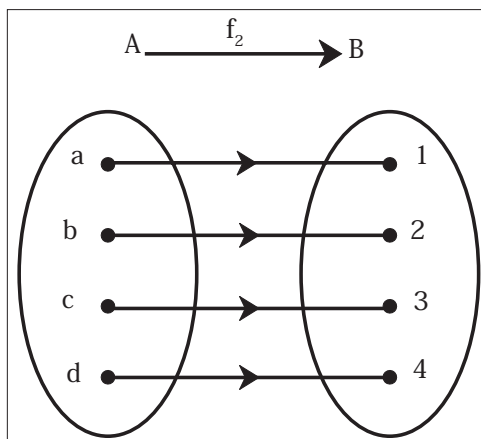
1)



**Fig. 2.4**

$f_1$  is one-one, not onto

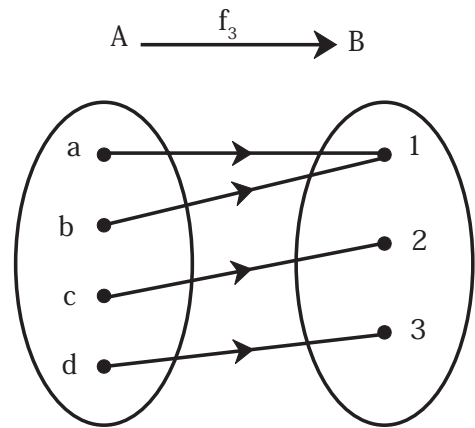
2)



**Fig. 2.5**

$f_2$  is one-one, onto

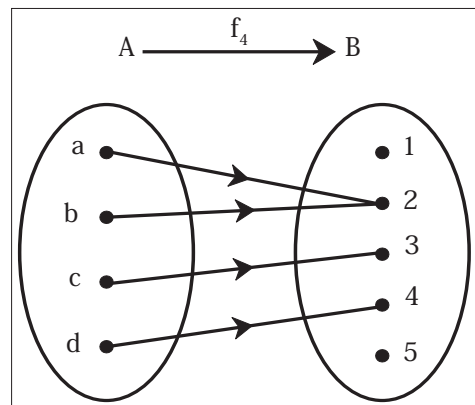
3)



**Fig. 2.6**

$f_3$  is not one-one, onto

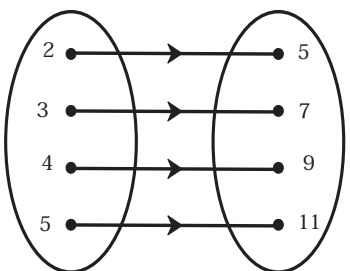
4)

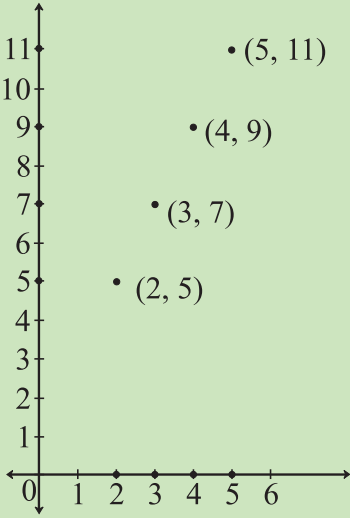


**Fig. 2.7**

$f_4$  is not one-one, not onto

### Representation of Function

Verbal form	Output is 1 more than twice the input Domain : Set of inputs Range : Set of outputs
Arrow form/ Venn Diagram Form	 <p><b>Fig. 2.7(A)</b> Domain : Set of pre-images Range: Set of images</p>

Ordered Pair (x, y)	$f = \{(2,5), (3,7), (4,9), (5,11)\}$ Domain : Set of 1 <sup>st</sup> components from ordered pair = $\{2, 3, 4, 5\}$ Range : Set of 2 <sup>nd</sup> components from ordered pair = $\{5, 7, 9, 11\}$										
Rule / Formula	$y = f(x) = 2x + 1$ Where $x \in N, 1 < x < 6$ $f(x)$ read as 'f of x' or 'function of x' Domain : Set of values of x, Range : Set of values of y for which $f(x)$ is defined										
Tabular Form	<table border="1"> <thead> <tr> <th>x</th><th>y</th></tr> </thead> <tbody> <tr> <td>2</td><td>5</td></tr> <tr> <td>3</td><td>7</td></tr> <tr> <td>4</td><td>9</td></tr> <tr> <td>5</td><td>11</td></tr> </tbody> </table> Domain : x values Range: y values	x	y	2	5	3	7	4	9	5	11
x	y										
2	5										
3	7										
4	9										
5	11										
Graphical form	 <p style="text-align: center;"><b>Fig. 2.7(B)</b></p> Domain: Extent of graph on x-axis. Range: Extent of graph on y-axis.										

### 2.1.2 Graph of a function:

If the domain of function is  $R$ , we can show the function by a graph in  $xy$  plane. The graph consists of points  $(x,y)$ , where  $y = f(x)$ .

### Evaluation of a function:

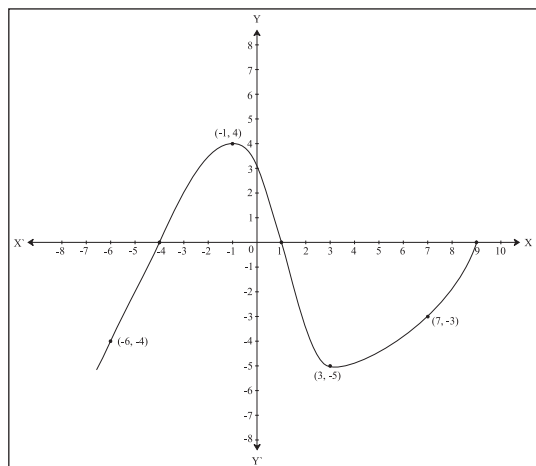
- 1) **Ex:** Evaluate  $f(x) = 2x^2 - 3x + 4$  at  $x = 7$  and  $x = -2t$

**Solution :**  $f(x)$  at  $x = 7$  is  $f(7)$

$$\begin{aligned}
 f(7) &= 2(7)^2 - 3(7) + 4 \\
 &= 2(49) - 21 + 4 \\
 &= 98 - 21 + 4 \\
 &= 81
 \end{aligned}$$

$$\begin{aligned}
 f(-2t) &= 2(-2t)^2 - 3(-2t) + 4 \\
 &= 2(4t^2) + 6t + 4 \\
 &= 8t^2 + 6t + 4
 \end{aligned}$$

- 2) Using the graph of  $y = g(x)$ , find  $g(-4)$  and  $g(3)$



**Fig. 2.8**

From graph when  $x = -4$ ,  $y = 0$  so  $g(-4) = 0$

From graph when  $x = 3$ ,  $y = -5$  so  $g(3) = -5$

- 3) If  $f(x) = 3x^2 - x$  and  $f(m) = 4$ , then find  $m$

**Solution :** As

$$\begin{aligned}
 f(m) &= 4 \\
 3m^2 - m &= 4 \\
 3m^2 - m - 4 &= 0 \\
 3m^2 - 4m + 3m - 4 &= 0 \\
 m(3m - 4) + 1(3m - 4) &= 0
 \end{aligned}$$

$$(3m - 4)(m + 1) = 0$$

Therefore,  $m = \frac{4}{3}$  or  $m = -1$

- 4) From the graph below find  $x$  for which  $f(x) = 4$

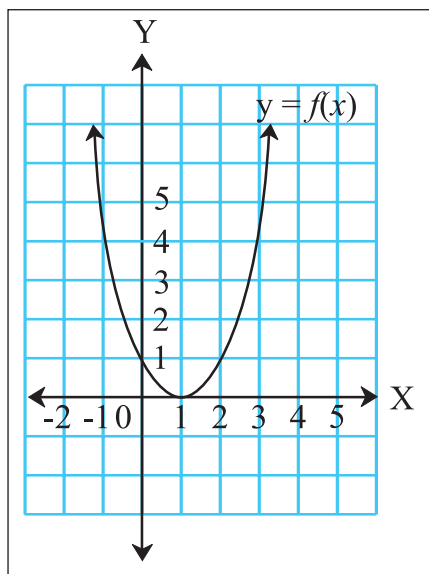


Fig. 2.9

**Solution :** To solve  $f(x) = 4$

find the values of  $x$  where graph intersects line  $y = 4$

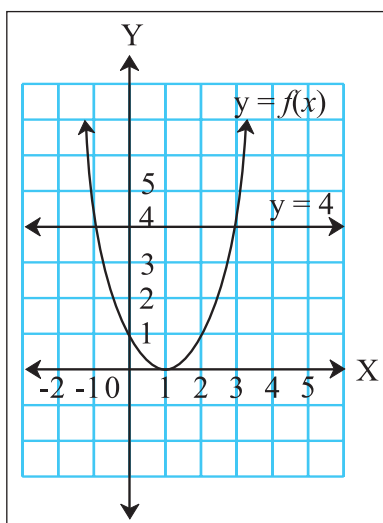


Fig. 2.10

Therefore,  $x = -1$  and  $x = 3$  are the values of  $x$  for which  $f(x) = 4$

- 5) From the equation  $4x + 7y = 1$  express

- $y$  as a function of  $x$
- $x$  as a function of  $y$

**Solution :** Given relation is  $4x + 7y = 1$

- i) From the given equation

$$7y = 1 - 4x$$

$$y = \frac{1-4x}{7} = \text{function of } x$$

$$\text{So } y = f(x) = \frac{1-4x}{7}$$

- ii) From the given equation

$$4x = 1 - 7y$$

$$x = \frac{1-7y}{4} = \text{function of } y$$

$$\text{So } x = g(y) = \frac{1-7y}{4}$$

### Some Fundamental Functions

#### 1) Constant Function :

$f: \mathbb{R} \rightarrow \mathbb{R}$  given by

$f(x) = k$ ,  $x \in \mathbb{R}$ ,  $k$  is constant is called the constant function.

**Example:**  $f(x) = 3$ ,  $x \in \mathbb{R}$

Graph of  $f(x) = 3$

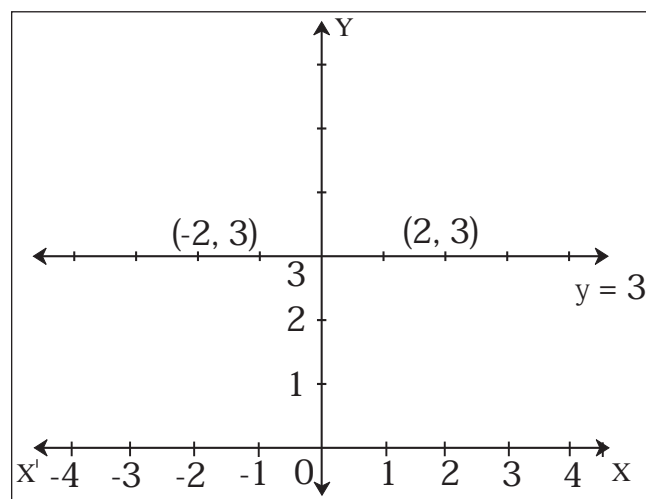


Fig. 2.11

**Domain :**  $\mathbb{R}$  or  $(-\infty, \infty)$  and **Range :**  $\{3\}$

## 2) Identity function

If the domain and co-domain are same as  $A$ , then we define a special function as identity function defined by  $f(x) = x$ , for every  $x \in A$ .

Let  $A = \mathbb{R}$ . Then identity function  $f: \mathbb{R} \rightarrow \mathbb{R}$ , where  $y = x$  is shown in the graph in fig. 2.12.

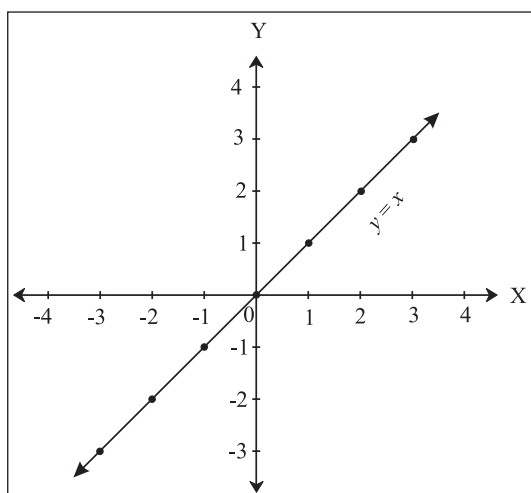


Fig. 2.12

Graph of  $f(x) = x$

**Domain :**  $\mathbb{R}$  or  $(-\infty, \infty)$  and **Range :**  $\mathbb{R}$  or  $(-\infty, \infty)$

## 3) Linear Function :

**Example :** A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  of the form  $f(x) = ax + b$

For example,  $f(x) = -2x + 3$ , the graph of which is shown in Fig. 2.13

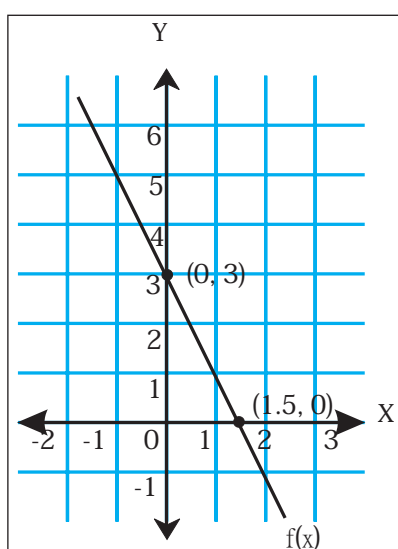


Fig. 2.13

**Domain :**  $\mathbb{R}$  or  $(-\infty, \infty)$  and **Range :**  $\mathbb{R}$  or  $(-\infty, \infty)$

**Properties :**

- 1) Graph of  $f(x) = ax + b$  is a line with slope ' $a$ ', y-intercept ' $b$ ' and x-intercept  $\left(-\frac{b}{a}\right)$ .
- 2) Graph is increasing when slope is positive and decreasing when slope is negative.

## 4) Quadratic Function

Function of the form  $f(x) = ax^2 + bx + c$  ( $a \neq 0$ )

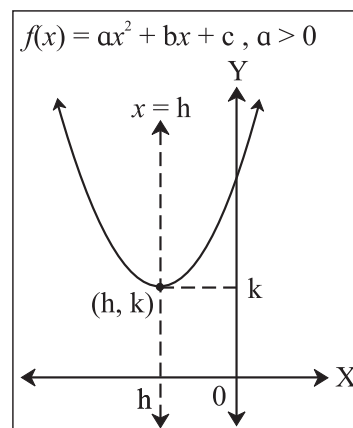


Fig. 2.14

**Domain :**  $\mathbb{R}$  or  $(-\infty, \infty)$  and **Range :**  $[k, \infty)$

**5) Function of the form  $f(x) = ax^n$ ,  $n \in \mathbb{N}$**   
(Note that this function is a multiple of power of  $x$ )

**i) Square Function**

**Example :**  $f(x) = x^2$

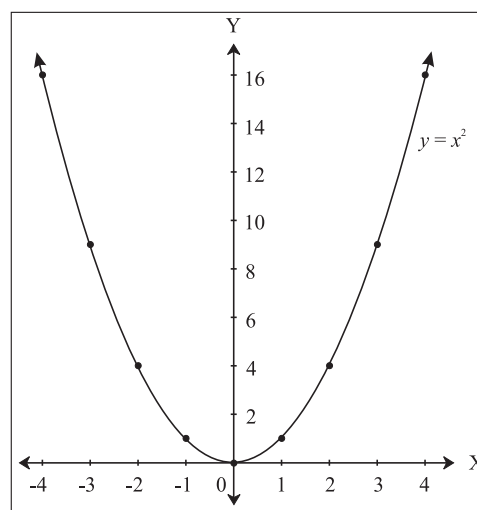
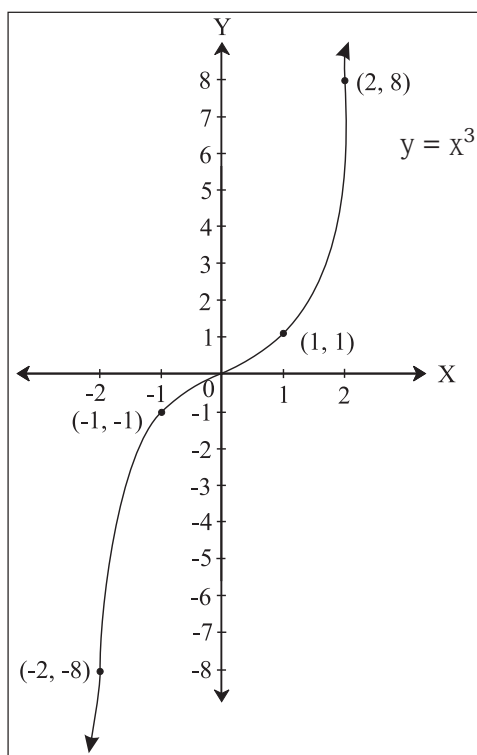


Fig. 2.15

**Domain :**  $\mathbb{R}$  or  $(-\infty, \infty)$  and **Range :**  $[0, \infty)$

## ii) Cube Function

**Example :**  $f(x) = x^3$



**Fig. 2.16**

**Domain :**  $\mathbb{R}$  or  $(-\infty, \infty)$  and **Range :**  $\mathbb{R}$  or  $(-\infty, \infty)$

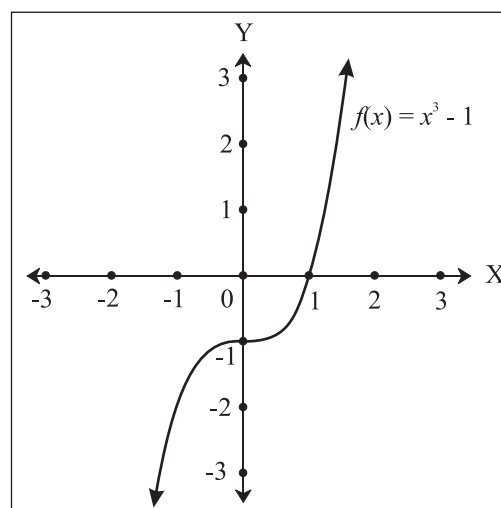
**6) Polynomial Function :** A function of the form

$$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n$$

is polynomial function of degree  $n$ , if  $a_0 \neq 0$ , and  $a_0, a_1, a_2, \dots, a_n$  are real. The graph of a general polynomial is more complicated and depends upon its individual terms.

Graph of  $f(x) = x^3 - 1$

$f(x) = (x - 1)(x^2 + x + 1)$  cuts  $x$ -axis at only one point  $(1, 0)$ , which means  $f(x)$  has one real root and two complex roots.



**Fig. 2.17**

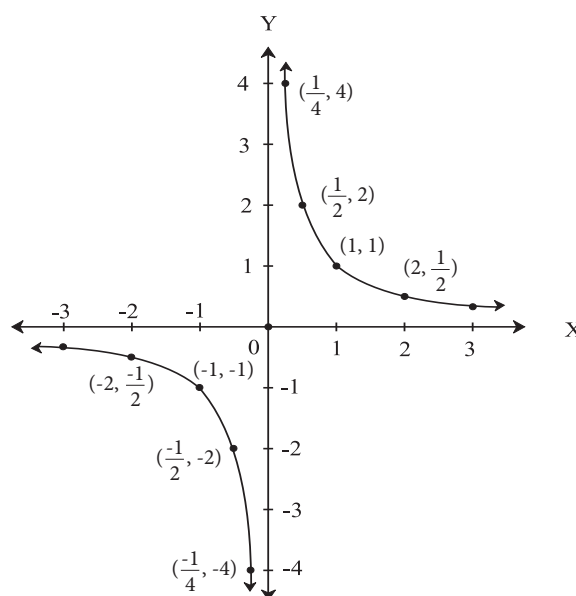
Note that, any polynomial of odd degree must have at least one real root, because the complex roots appear in conjugate pairs.

## 7) Rational Function

**Definition:** Given polynomials  $p(x)$ ,  $q(x)$

$f(x) = \frac{p(x)}{q(x)}$  ( $q(x) \neq 0$ ) is called a rational function.

For example,  $f(x) = \frac{1}{x}$



**Fig. 2.18**

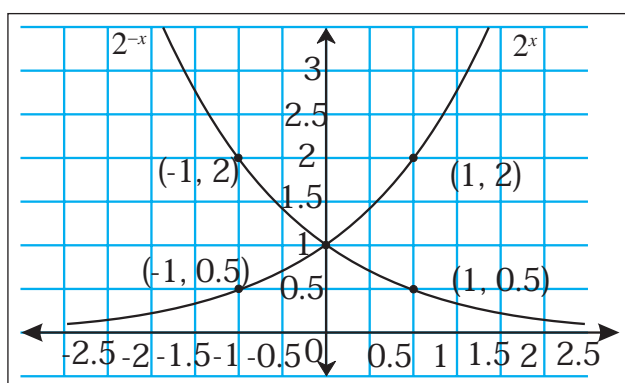
**Domain :**  $\mathbb{R} - \{0\}$  and **Range :**  $\mathbb{R} - \{0\}$

**Example :** Domain of function  $f(x) = \frac{2x+9}{x^3-25x}$

i.e.  $f(x) = \frac{2x+9}{x(x-5)(x+5)}$  is  $\mathbb{R} - \{0, 5, -5\}$  as for

$x = 0$ ,  $x = -5$  and  $x = 5$ , denominator becomes 0 .

**8) Exponential Function :** A function  $f: \mathbb{R} \rightarrow \mathbb{R}^+$  given by  $f(x) = a^x$  is an exponential function with base  $a$  and exponent (or index)  $x$ ,  $a \neq 1$ ,  $a > 0$  and  $x \in \mathbb{R}$ .



**Fig. 2.19**

**Domain:**  $\mathbb{R}$  and **Range :**  $(0, \infty) = \mathbb{R}^+$

**9) Logarithmic Function:** A function  $f: \mathbb{R}^+ \rightarrow \mathbb{R}$ , defined as  $f(x) = \log_a x$ , ( $a > 0$ ,  $a \neq 1$ ,  $x > 0$ ) where  $y = \log_a x$  if and only if  $x = a^y$  is called Logarithmic Function.

$y = \log_a x$  is equivalent to  $a^y = x$ .  
logarithmic form  exponential form

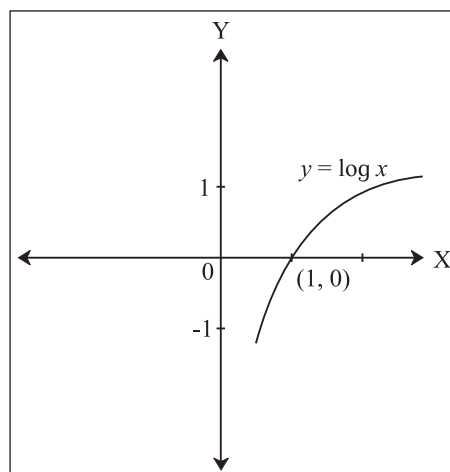


- 1)  $y = \log_a x \Leftrightarrow a^y = x \Leftrightarrow \text{anti } \log_a y = x$
- 2) As  $a^0 = 1$ , so  $\log_a 1 = 0$  and as  $a^1 = a$ , so  $\log_a a = 1$

3) As  $a^x = a^y \Leftrightarrow x = y$  so  $\log_a x = \log_a y \Leftrightarrow x = y$

4) For natural base  $e$ ,  $\log_e x = \ln x$  ( $x > 0$ ) is called as Natural Logarithm Function.

**Domain :**  $(0, \infty)$



**Fig. 2.20**

**Range :**  $(-\infty, \infty)$

**Rules of Logarithm :**

- 1)  $\log_m ab = \log_m a + \log_m b$
- 2)  $\log_m \frac{a}{b} = \log_m a - \log_m b$
- 3)  $\log_m a^b = b \cdot \log_m a$
- 4) Change of base formula:

For  $a, x, b > 0$  and  $a, b \neq 1$ ,  $\log_a x = \frac{\log_b x}{\log_b a}$

**Note:**  $\log_a x = \frac{\log_x x}{\log_x a} = \frac{1}{\log_x a}$

**Algebra of functions:**

If  $f$  and  $g$  are functions from  $X \rightarrow \mathbb{R}$ , then the functions  $f + g, f - g, fg, \frac{f}{g}$  are defined as follows.

Operations
$(f + g)(x) = f(x) + g(x)$
$(f - g)(x) = f(x) - g(x)$
$(f \cdot g)(x) = f(x) \cdot g(x)$
$\frac{f}{g}(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$

**Ex.** If  $f(x) = x^2 + 2$  and  $g(x) = 5x - 8$ , then find

i)  $(f + g)(1)$

ii)  $(f - g)(-2)$

iii)  $(f \cdot g)(3m)$

iv)  $\frac{f}{g}(0)$

i) As  $(f + g)(x) = f(x) + g(x)$

$$(f + g)(1) = f(1) + g(1)$$

$$= (1)^2 + 2 + 5(1) - 8$$

$$= 1 + 2 + 5 - 8$$

$$= 8 - 8$$

$$= 0$$

ii) As  $(f - g)(x) = f(x) - g(x)$

$$(f - g)(-2) = f(-2) - g(-2)$$

$$= [(-2)^2 + 2] - [5(-2) - 8]$$

$$= [4 + 2] - [-10 - 8]$$

$$= 6 + 18$$

$$= 24$$

iii) As  $(fg)(x) = f(x) g(x)$

$$(f \cdot g)(3m) = f(3m) g(3m)$$

$$= [(3m)^2 + 2] [5(3m) - 8]$$

$$= [9m^2 + 2] [15m - 8]$$

$$= 135m^3 - 72m^2 + 30m - 16$$

iv) As  $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$

$$\frac{f}{g}(0) = \frac{f(0)}{g(0)}$$

$$= \frac{2}{-8}$$

$$= -\frac{1}{4}$$

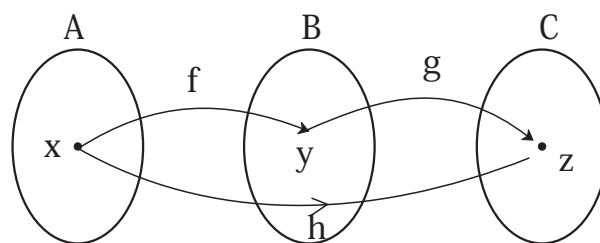
### 2.1.3 Composite function:

Let  $f:A \rightarrow B$  and  $g:B \rightarrow C$  where  $f(x) = y$  and  $g(y) = z$ ,  $x \in A$ ,  $y \in B$ ,  $z \in C$ . We define a function  $h:A \rightarrow C$  such that  $h(x) = z$  then the function  $h$  is called composite function of  $f$  and  $g$  and is denoted by  $g \circ f$ .

$$\therefore g \circ f : A \rightarrow C$$

$$g \circ f(x) = g[f(x)]$$

It is also called function of a function.



**Fig. 2.21**

e.g.

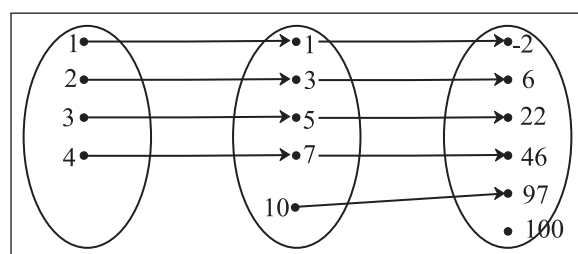
Let  $A = \{1, 2, 3, 4\}$  and  $B = \{1, 3, 5, 7, 10\}$

$C = \{-2, 6, 22, 46, 97, 100\}$  where

$f = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$  and

$g = \{(1, -2), (3, 6), (5, 22), (7, 46), (10, 97)\}$

$\therefore g \circ f = \{(1, -2), (2, 6), (3, 22), (4, 46)\}$



**Fig. 2.22**



Since

$$f = \{(x, y) / x \in A, y \in B, \text{ and } y = 2x - 1\}$$

$$g = \{(y, z) / y \in B, z \in C, \text{ and } z = y^2 - 3\}$$

then

$$g \circ f(x) = \{(x, z) / x \in A, z \in C\} \text{ and}$$

$$z = (2x - 1)^2 - 3$$

**Ex 1.** If  $f(x) = \frac{2}{x+5}$  and  $g(x) = x^2 - 1$ , then find

i)  $(f \circ g)(x)$  ii)  $(g \circ f)(3)$

**Solution :**

i) As  $(f \circ g)(x) = f[g(x)]$  and  $f(x) = \frac{2}{x+5}$

Replace  $x$  from  $f(x)$  by  $g(x)$ , to get

$$\begin{aligned} (f \circ g)(x) &= \frac{2}{g(x)+5} \\ &= \frac{2}{x^2-1+5} \\ &= \frac{2}{x^2+4} \end{aligned}$$

ii) As  $(g \circ f)(x) = g[f(x)]$  and  $g(x) = x^2 - 1$

Replace  $x$  by  $f(x)$  in  $g(x)$ , to get

$$\begin{aligned} (g \circ f)(x) &= [f(x)]^2 - 1 \\ &= \left(\frac{2}{x+5}\right)^2 - 1 \end{aligned}$$

Now let  $x = 3$

$$\begin{aligned} (g \circ f)(3) &= \left(\frac{2}{3+5}\right)^2 - 1 \\ &= \left(\frac{2}{8}\right)^2 - 1 \\ &= \left(\frac{1}{4}\right)^2 - 1 \\ &= \frac{1-16}{16} \\ &= -\frac{15}{16} \end{aligned}$$

**Ex 2.** If  $f(x) = x^2$ ,  $g(x) = x + 5$ , and  $h(x) = \frac{1}{x}$ , find  $(g \circ f \circ h)(x)$

→ As  $(g \circ f \circ h)(x) = g\{f[h(x)]\}$  and

$$g(x) = x + 5$$

Replace  $x$  in  $g(x)$  by  $f[h(x)] + 5$  to get

$$(g \circ f \circ h)(x) = f[h(x)] + 5$$

Now  $f(x) = x^2$ , so replace  $x$  in  $f(x)$  by  $h(x)$ , to get

$$(g \circ f \circ h)(x) = f[h(x)]^2 + 5$$

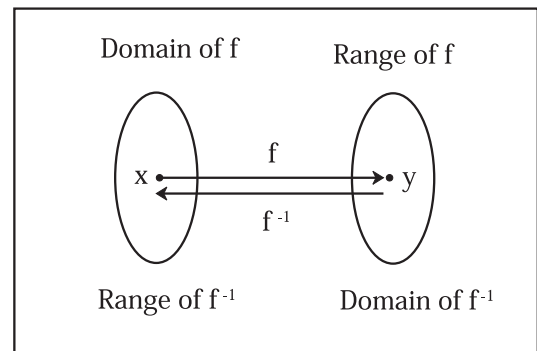
Now  $h(x) = \frac{1}{x}$  therefore,

$$\begin{aligned} (g \circ f \circ h)(x) &= \left(\frac{1}{x}\right)^2 + 5 \\ &= \frac{1}{x^2} + 5 \end{aligned}$$

#### 2.1.4 : Inverse functions:

Let  $f: A \rightarrow B$  be one-one and onto function and  $f(x) = y$  for  $x \in A$ . The inverse function

$f^{-1}: B \rightarrow A$  is defined as  $f^{-1}(y) = x$  for  $y \in B$ .



**Fig. 2.23**

**Note:** As  $f$  is one-one and onto every element  $y \in B$  has a unique element  $x \in A$  associated with  $y$ . Also note that  $f \circ f^{-1}(x) = x$

**Ex. 1** If  $f$  is one-one onto function with  $f(3) = 7$  then  $f^{-1}(7) = 3$ .

**Ex. 2.** If  $f$  is one-one onto function with

$$f(x) = 9 - 5x, \text{ find } f^{-1}(-1).$$

→ Let  $f^{-1}(-1) = m$ , then  $-1 = f(m)$

Therefore,

$$-1 = 9 - 5m$$

$$5m = 9 + 1$$

$$5m = 10$$

$$m = 2$$

That is  $f(2) = -1$ , so  $f^{-1}(-1) = 2$ .

**Ex.3** Verify that  $f(x) = \frac{x-5}{8}$  and  $g(x) = 8x + 5$  are inverse functions.

As  $f(x) = \frac{x-5}{8}$ , replace  $x$  in  $f(x)$  with  $g(x)$

$$\begin{aligned} f[g(x)] &= \frac{g(x)-5}{8} \\ &= \frac{8x+5-5}{8} \\ &= \frac{8x}{8} \\ &= x \end{aligned}$$

and  $g(x) = 8x + 5$ , replace  $x$  in  $g(x)$  with  $f(x)$

$$\begin{aligned} g[f(x)] &= 8f(x) + 5 \\ &= 8\left[\frac{x-5}{8}\right] + 5 \\ &= x - 5 + 5 \\ &= x \end{aligned}$$

As  $f[g(x)] = x$  and  $g[f(x)] = x$ ,  $f$  and  $g$  are inverse functions.

**Ex. 4:** Determine whether the function

$$f(x) = \frac{2x+1}{x-3} \text{ has inverse, if it exists find it.}$$

( $f^{-1}$  exists only if  $f$  is one-one and onto.)

**Solution :** Consider  $f(x_1) = f(x_2)$ ,

Therefore,

$$\frac{2x_1+1}{x_1-3} = \frac{2x_2+1}{x_2-3}$$

$$(2x_1+1)(x_2-3) = (2x_2+1)(x_1-3)$$

$$2x_1x_2 - 6x_1 + x_2 - 3 = 2x_1x_2 - 6x_2 + x_1 - 3$$

$$-6x_1 + x_2 = -6x_2 + x_1$$

$$7x_2 = 7x_1$$

$$x_2 = x_1$$

Hence,  $f$  is one-one function.

Let  $f(x) = y$ , so  $y = \frac{2x+1}{x-3}$

Express  $x$  as function of  $y$ , as follows

$$y = \frac{2x+1}{x-3}$$

$$y(x-3) = 2x+1$$

$$xy - 3y = 2x+1$$

$$xy - 2x = 3y+1$$

$$x(y-2) = 3y+1$$

$$x = \frac{3y+1}{y-2}$$

Here the range of  $f(x)$  is  $\mathbb{R} - \{2\}$ .

$x$  is defined for all  $y$  in the range.

Therefore  $f(x)$  is onto function.

As function is one-one and onto, so  $f^{-1}$  exists.

As  $f(x) = y$ , so  $f^{-1}(y) = x$

$$\text{Therefore, } f^{-1}(y) = \frac{3y+1}{y-2}$$

Replace  $x$  by  $y$ , to get

$$f^{-1}(x) = \frac{3x+1}{x-2}.$$

### 2.1.5 :Some Special Functions

**i) Signum function :** The function  $f: \mathbb{R} \rightarrow \mathbb{R}$

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

is called the signum function. It is denoted as  $\text{sgn}(x)$

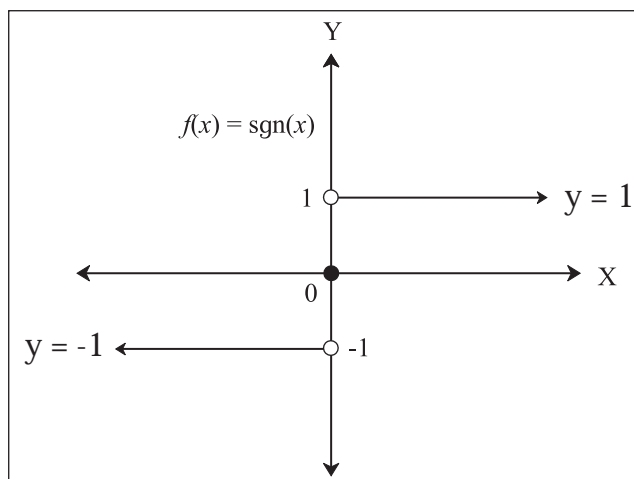


Fig. 2.24

**Domain:**  $\mathbb{R}$  **Range:**  $\{-1, 0, 1\}$

## ii) Absolute value function (Modulus function):

**Definition:** Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = |x|$ ,  $\forall x \in \mathbb{R}$

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

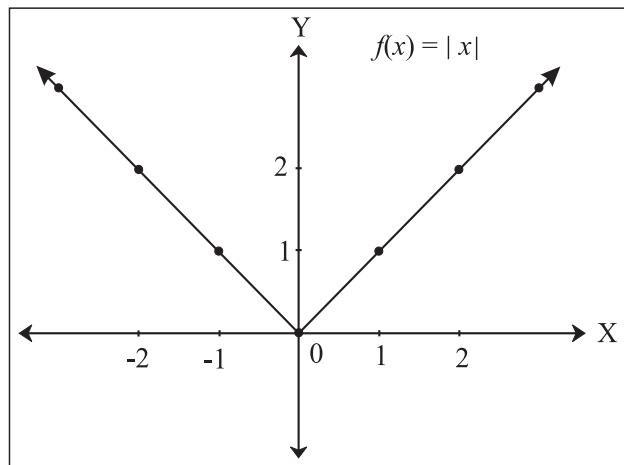


Fig. 2.25

**Domain :**  $\mathbb{R}$  or  $(-\infty, \infty)$  and **Range :**  $[0, \infty)$

## iii) Greatest Integer Function (Step Function):

**Definition:** For every real  $x$ ,  $f(x) = [x]$  = The greatest integer less than or equal to  $x$ .

$$f(x) = n, \text{ if } n \leq x < n + 1, x \in [n, n + 1)$$

We have

$$f(x) = \begin{cases} -2 & \text{if } -2 \leq x < -1 \text{ or } x \in [-2, -1) \\ -1 & \text{if } -1 \leq x < 0 \text{ or } x \in [-1, 0) \\ 0 & \text{if } 0 \leq x < 1 \text{ or } x \in [0, 1) \\ 1 & \text{if } 1 \leq x < 2 \text{ or } x \in [1, 2) \\ 2 & \text{if } 2 \leq x < 3 \text{ or } x \in [2, 3) \end{cases}$$

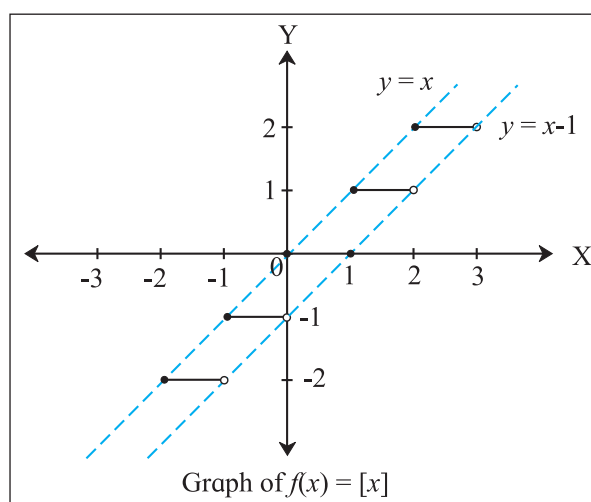


Fig. 2.26

**Domain** =  $\mathbb{R}$  and **Range** =  $\mathbb{I}$  (Set of integers)

## EXERCISE 2.1

1) Check if the following relations are functions.

(a)

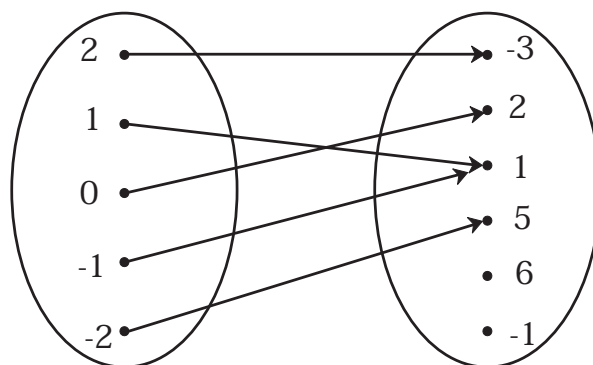


Fig. 2.27

(b)

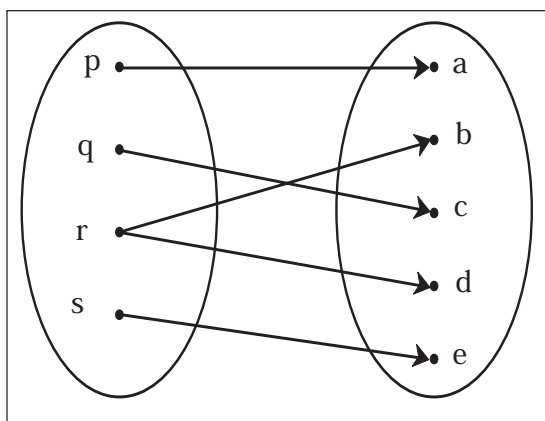


Fig. 2.28

(c)

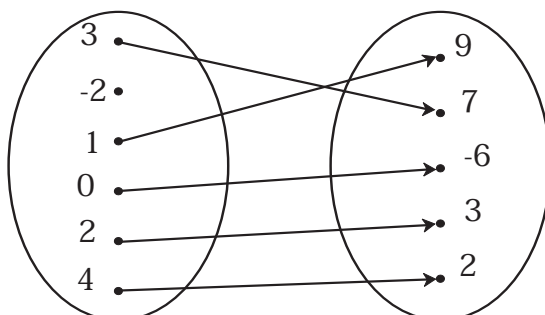


Fig. 2.29

- 2) Which sets of ordered pairs represent functions from  $A = \{1, 2, 3, 4\}$  to  $B = \{-1, 0, 1, 2, 3\}$ ? Justify.

- (a)  $\{(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)\}$   
(b)  $\{(1, 2), (2, -1), (3, 1), (4, 3)\}$   
(c)  $\{(1, 3), (4, 1), (2, 2)\}$   
(d)  $\{(1, 1), (2, 1), (3, 1), (4, 1)\}$

- 3) If  $f(m) = m^2 - 3m + 1$ , find

- (a)  $f(0)$  (b)  $f(-3)$   
(c)  $f\left(\frac{1}{2}\right)$  (d)  $f(x+1)$   
(e)  $f(-x)$

- 4) Find  $x$ , if  $g(x) = 0$  where

(a)  $g(x) = \frac{5x-6}{7}$  (b)  $g(x) = \frac{18-2x^2}{7}$

(c)  $g(x) = 6x^2 + x - 2$

- 5) Find  $x$ , if  $f(x) = g(x)$  where

$f(x) = x^4 + 2x^2$ ,  $g(x) = 11x^2$

- 6) If  $f(x) = \begin{cases} x^2 + 3, & x \leq 2 \\ 5x + 7, & x > 2 \end{cases}$ , then find

- (a)  $f(3)$  (b)  $f(2)$  (c)  $f(0)$

- 7) If  $f(x) = \begin{cases} 4x - 2, & x \leq -3 \\ 5, & -3 < x < 3 \\ x^2, & x \geq 3 \end{cases}$ , then find

- (a)  $f(-4)$  (b)  $f(-3)$   
(c)  $f(1)$  (d)  $f(5)$

- 8) If  $f(x) = 3x + 5$ ,  $g(x) = 6x - 1$ , then find

- (a)  $(f+g)(x)$  (b)  $(f-g)(2)$   
(c)  $(fg)(3)$  (d)  $(f/g)(x)$  and its domain.

- 9) If  $f(x) = 2x^2 + 3$ ,  $g(x) = 5x - 2$ , then find

- (a)  $f \circ g$  (b)  $g \circ f$   
(c)  $f \circ f$  (d)  $g \circ g$



**Let's remember!**

- A Function from set A to the set B is a relation which associates every element of set A to unique element of set B and is denoted by  $f: A \rightarrow B$ .
- If  $f$  is a function from set A to the set B and if  $(x, y) \in f$  then  $y$  is called the image of  $x$  under  $f$  and  $x$  is called the pre-image of  $y$  under  $f$ .



- A function  $f: A \rightarrow B$  is said to be one-one if distinct elements in  $A$  have distinct images in  $B$ .
- A function  $f: A \rightarrow B$  is said to be onto if every element of  $B$  is image of some element of  $A$  under the function  $f$ .
- A function  $f: A \rightarrow B$  is such that there exists atleast one element in  $B$  which does not have pre-image in  $A$  then  $f$  is said to be an into function.
- If  $f: A \rightarrow B$  and  $g: B \rightarrow C$  with  $f(x) = y$  and  $g(y) = z$ ,  $x \in A$ ,  $y \in B$ ,  $z \in C$  then define  $h: A \rightarrow C$  such that  $h(x) = z$ , then the function  $h$  is called composite function of  $f$  and  $g$  and is denoted by  $gof$ .
- If  $f: A \rightarrow B$  is one-one and onto,  $g: B \rightarrow A$  is one-one and onto such that  $gof: A \rightarrow A$  and  $fog: B \rightarrow B$  are both identity functions then  $f$  and  $g$  are inverse functions of each other.
- Domain of  $f$  = Range of  $f^{-1}$   
Range of  $f$  = Domain of  $f^{-1}$

### MISCELLANEOUS EXERCISE -2

- Which of the following relations are functions? If it is a function determine its domain and range.
  - $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$
  - $\{(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)\}$
  - $\{(1, 1), (3, 1), (5, 2)\}$
- A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{3x}{5} + 2$ ,  $x \in \mathbb{R}$ . Show that  $f$  is one-one and onto. Hence find  $f^{-1}$ .
- A function  $f$  is defined as follows  
 $f(x) = 4x + 5$ , for  $-4 \leq x < 0$ .

Find the values of  $f(-1)$ ,  $f(-2)$ ,  $f(0)$ , if they exist.

- A function  $f$  is defined as follows:  
 $f(x) = 5 - x$  for  $0 \leq x \leq 4$   
Find the value of  $x$  such that  $f(x) = 3$
- If  $f(x) = 3x^2 - 5x + 7$  find  $f(x - 1)$ .
- If  $f(x) = 3x + a$  and  $f(1) = 7$  find  $a$  and  $f(4)$ .
- If  $f(x) = ax^2 + bx + 2$  and  $f(1) = 3$ ,  $f(4) = 42$ . find  $a$  and  $b$ .
- If  $f(x) = \frac{2x-1}{5x-2}$ ,  $x \neq \frac{2}{5}$   
Verify whether  $(f \circ f)(x) = x$ .
- If  $f(x) = \frac{x+3}{4x-5}$ ,  $g(x) = \frac{3+5x}{4x-1}$   
then verify that  $(f \circ g)(x) = x$ .

### ACTIVITIES

#### Activity 2.1 :

If  $f(x) = \frac{x+3}{x-2}$ ,  $g(x) = \frac{2x+3}{x-1}$  Verify whether  $f \circ g(x) = g \circ f(x)$ .

#### Activity 2.2 :

$f(x) = 3x^2 - 4x + 2$ ,  $n \in \{0, 1, 2, 3, 4\}$  then represent the function as

- By arrow diagram
- Set of ordered pairs
- In tabular form
- In graphical form

#### Activity 2.3 :

If  $f(x) = 5x - 2$ ,  $x > 0$ , find  $f^{-1}(x)$ ,  $f^{-1}(7)$ , for what value of  $x$  is  $f(x) = 0$ .

